A Global Modeling Approach Using Interpolating Wavelets

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Abstract — A MESFET and a two-dimensional cavity enclosing a cylinder are simulated using a non-uniform mesh generated by an interpolating wavelet scheme. A self-adaptive mesh is implemented and controlled by the wavelet coefficient threshold. A fine mesh can therefore be used in domains where the unknown quantities are varying rapidly and a coarse mesh can be used where the unknowns are varying slowly. It is shown that good accuracy can be achieved while compressing the number of unknowns by 50 to 80 % during the whole simulation. This represents the on going effort toward a numerical technique that uses wavelets to solve both Maxwell's equations and the semiconductor equations. Such a method is of great interest to deal with the multi-scale problem that is the full wave simulation of an active microwave circuits.

I. Introduction

With the increasing flow of data in the telecommunication world, the performances of high-frequency devices have become more demanding. Typical circuit simulators no longer represent the accurate tool to characterize microwave circuits. Electromagnetic simulators need to be used to tackle the problems of EM interference such as packaging effects and coupling between sub-circuits among others. The full wave analysis of microwave circuits (Global Modeling) is a tremendous task that requires involved numerical techniques and algorithms [1]. It is to this date, unsuitable for circuit optimizations and design. However there is the need to develop a numerical technique that would allow global modeling simulation to be used while designing circuits. Due to the different scales of the active and passive parts of a circuit, there is a need for a numerical technique that would adaptively refine the mesh where it is needed. Such a technique corresponds to a multiresolution analysis of the problem. A very attractive way of implementing a multiresolution analysis is to use wavelets [2]. It was demonstrated [3] that finite difference schemes can be derived using the method of moments with wavelet expansions of the fields. The resulting numerical technique has been called the Multiresolution Time Domain Technique (MRTD)[4]. For non-linear equations such as semiconductor modeling equations, wavelet Galerkin method could become quite time consuming [5]. Therefore a different wavelet approach need to be used to solve the equations characterizing the motion of the electrons inside the active device.

We propose to generate a non-uniform mesh using an interpolating wavelet scheme. The wavelet coefficients are directly related to the physical domain as they represent the error between the exact solution on the grid and the interpolated value from the previous level. This scheme allows us to multiply and differentiate very quickly. We will follow the algorithm explained in [6] that has been used to solve 1-D Maxwell's equations in [7] and a 2-D PN junction in [8]. We will use this scheme to solve a two-dimensional cavity (TE case) enclosing a cylinder representing a discontinuity inside the cavity. This cylinder will allow us to see the refinement process that occurs during the simulation. We will also present the simulation of a typical MESFET.

II. Two dimensional cavity

The technique used was originally presented in [9]. A set of dyadic grids that defines different resolution levels is considered. A coarsest grid is defined, then a finer grid is generated by introducing points half way between those of the coarse grid. Values at odd grid points are kept unchanged while values at even points are interpolated by a polynomial of order three ($p=4$) or order one ($p=2$). The so-called sparse point representation (SPR) of a function f [6] can be created. We start with the coarsest grid, we create an irregular mesh by computing the error between the exact value of f on the coarse grid and the value

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obtained by interpolation. The error is the wavelet coefficient. It carries the detailed signal. By removing the points that can be interpolated, we greatly compress the data. The compression depends on the threshold value set as the minimum error for the interpolation procedure. All the computations are then done on the irregular mesh.

A 2-D cavity was simulated according to the example presented in [10]. The cavity is discretized by a mesh of 65*33 with a space increment *dx=3.0 mm*. The space increment is *dt=5.0 ps*. Fig. 1 shows the y component of the electric field at two different times.

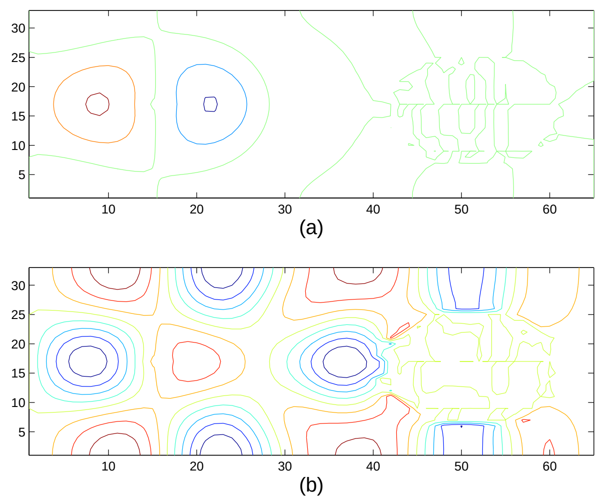

Fig. 1. Electric field in the y direction, (a) t=520 ps, (b) t=840 ps.

We can see that at t=520 ps the wave did not reach the cylinder yet, whereas at t=840 ps, the modes inside the cavity are developing and scattering due to the cylinder is occurring.

Fig. 2 shows the sparse point representation or in other terms, the non-uniform mesh generated by the scheme at the same times than Fig.1. This demonstrates the self-adaptibility of the mesh which gets finer in regions where the fields are varying rapidly. At t=840 ps, scattering due to the cylinder needs to be modeled accurately therefore more points are used around the cylinder.

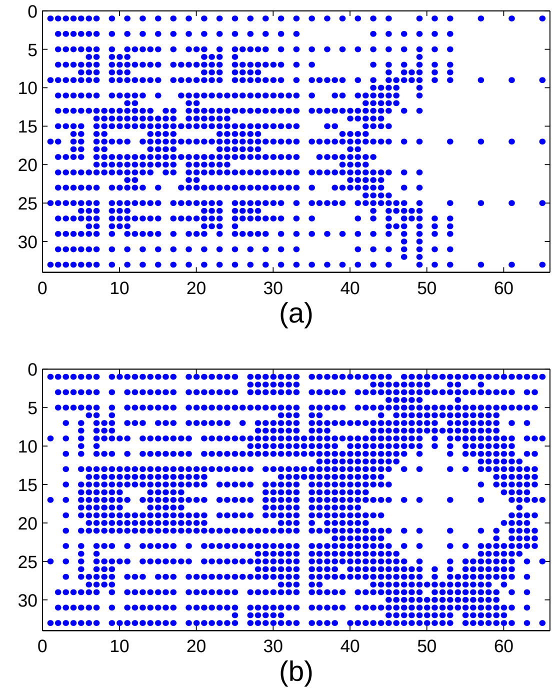

Fig. 2. Sparse point representation of the y component of the electric field at (a) t=520 ps and (b) t=840 ps

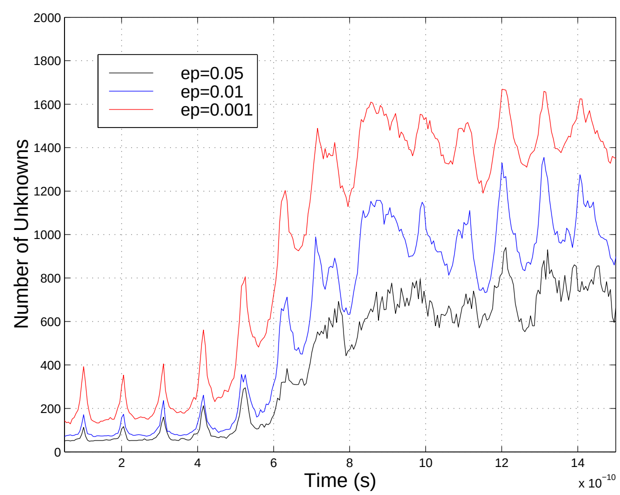

Fig. 3. Number of unknowns remaining in the SPR for three different value of wavelet threshold.

At every time step this non-uniform mesh is generated by the wavelet scheme. Thus the number of unknowns varies in time. Fig. 3 shows this behavior. Three wavelet thresholds are used. We can see that as the threshold gets smaller, more points need to be used in the mesh. The error computing during the interpolation needs to be smaller thus finer mesh are used.

III. MESFET SIMULATION

In order to simulate sub-micrometer gate length transistors that are used at high frequencies, a full hydrodynamic model must be used.

This model is based on the moments of Boltzmann's transport equations obtained by integration over the momentum space. Three equations need to be solved together with Poisson's equations in order to get the quasi-static characteristics of the transistor. This system of coupled highly nonlinear partial differential equations is as follows: current continuity (1), energy conservation (2) and momentum conservation for the x-component (3).

$$\frac{\partial n}{\partial t} + \nabla.(n\bar{v}) = 0 \quad (1)$$

$$\frac{\partial (n\varepsilon)}{\partial t} + qn\bar{v}.\bar{E} + \nabla.(n\bar{v}(\varepsilon + k_b T)) = -\frac{n(\varepsilon - \varepsilon_0)}{\tau_\varepsilon(\varepsilon)} \quad (2)$$

$$\frac{\partial (nm^* v_x)}{\partial t} + qnE_x + \nabla.(nm^* v_x \bar{v}) + \frac{\partial (nk_b T)}{\partial x} = \frac{nm^* v_x}{\tau_m(\varepsilon)} \quad (3)$$

Where n is the carrier density, v the velocity, ε the energy, E the electric field, m^* the effective mass, k_b the Boltzmann's constant, T the temperature and τ_m , τ_ε the momentum and the energy relaxation time.

The solution of this system of partial differential equations represents the complete hydrodynamic model. Simplified models are obtained by neglecting time and/or space derivatives, the so-called inertia effects, in the momentum equation (3).

At the early stage of this work, a drift diffusion model is derived by assuming that the momentum is given by (4). Where the momentum relaxation time is obtained assuming that the mobility is a function of the electric field (5).

$$qE_x + \frac{1}{n}\frac{\partial (nk_b T)}{\partial x} = \frac{m^* v_x}{\tau_m(\varepsilon)} \quad (4)$$

$$\mu(E) = \mu_0.\left(1 + \frac{\mu_0 |E|}{10^7}\right)^{-1} \quad (5)$$

A MESFET with the following dimensions is simulated: 0.6 *μm* gate length, 1 *μm* long source and drain electrodes, 1 *μm* source-gate gap, 1 *μm* gate-drain separation, 1.2 *μm* deep active layer. The doping of the active layer is 1.2×10^{17} *A/cm*^{3} and the buffer layer has an electron concentration of 1×10^{14} *cm*$^{-3}$.

In Fig. 4, the I-V characteristics of the simulated MESFET are shown. The MESFET exhibits conventional dc characteristics which demonstrates the potential of this scheme.

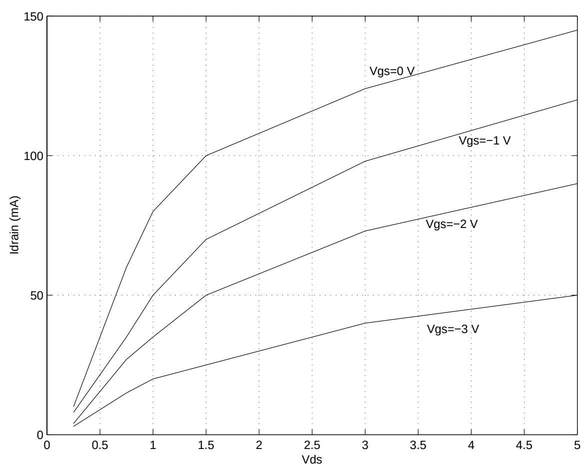

Fig. 4. I-V Characteristics of the simulated MESFET

As shown in Fig. 3, this scheme allows us to use different mesh depending on how the unknown quantities vary. At the beginning of the simulation the carrier density are initialized to the doping profile so the irregular mesh is really coarse, as time evolves the depletion region starts to be created and more points are needed to describe it.

This illustrates the dynamic behavior of the mesh. The SPR can be generated by wavelets of different order. Numerical simulation were performed using $p=4$ and $p=2$. Fig. 5 demonstrates that for $\varepsilon =0.01$ and $\varepsilon =0.001$, the linear interpolation achieves a better compression ratio than the cubic interpolation. For $\varepsilon =0.0001$ however, it is the opposite. It is our understanding that this breaking point is a function of the wavelet threshold, the bias point and the physical dimensions.

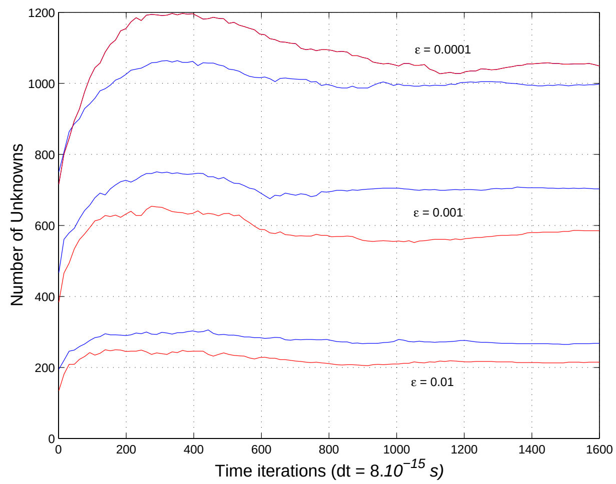

Fig. 5. Mesh Adaptability for three different values of wavelet threshold. (blue lines *p=2* , red lines *p=4*)

As the number of unknowns can be compressed this technique is expected to reduce the computation time. But the error must be kept small. In other terms the currents must be the same as the ones obtained from standard finite difference. Fig. 6, shows the relative error on the drain current for the three threshold values presented in Fig 5. We can see that for $\varepsilon = 0.001$ the error is around 4 %. As the threshold gets bigger the mesh is coarser and the error grows. Finally, good compression can be obtained together with good accuracy.

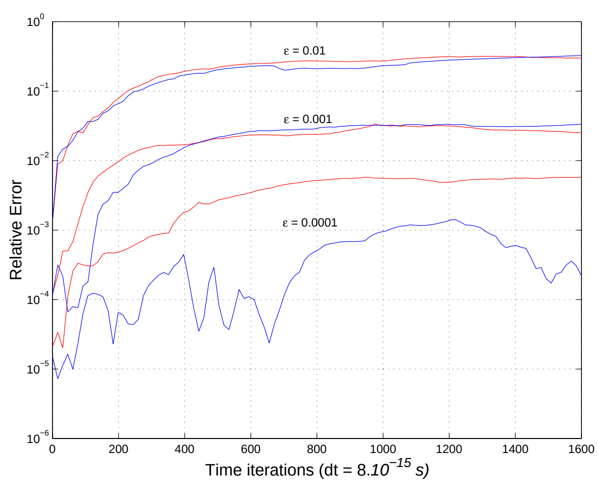

Fig. 6 Relative Error on the drain current for three different values of wavelet threshold. (blue lines *p=2* , red lines *p=4*)

IV. CONCLUSION

A non-uniform mesh is generated by a wavelet-based approach. The interpolation wavelet scheme used allows the implementation of a self-adaptive mesh in time. It is shown that the threshold coefficient controls the compression ratio and therefore the accuracy compared to a standard finite difference scheme. A MESFET has been simulated using this scheme, 83 % compression in the unknowns is obtained with an accuracy of 4 %. Computation time is expected to be reduced dramatically with this scheme. The same characteristics can be obtained for electromagnetic problems.

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